

A MARKOV MODEL DESCRIPTION OF CHANGEOVER PROBABILITIES ON CONCURRENT VARIABLE-INTERVAL SCHEDULES¹

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The primary data were peck-by-peck sequential records of four pigeons responding on several different concurrent variable-interval schedules. According to the hypothesis that the subject chooses the alternative with the highest probability of reinforcement at the moment, response-by-response performance in concurrent schedules should show sequential dependencies. However, such dependencies were not found, and it was possible to describe molecular-level performance with simple Markov chain models. The Markov model description implies that the momentary changeover probabilities were proportional to the overall relative reinforcement frequencies, and that changeover probabilities did not change as a function of previous responding. A second finding was that although a changeover-delay procedure was omitted, relative response frequencies closely approximated relative reinforcement frequencies.

Key words: concurrent variable-interval schedules, matching, molecular analysis, molar analysis, changeover probability, changeover delay, key peck, pigeons

Descriptions of performance on concurrent variable-interval variable-interval (*conc VI VI*) schedules typically focus on the relationship between the overall relative response rate and the overall relative reinforcement rate (e.g., Herrnstein, 1970). This is called a molar-level description (e.g., Shimp, 1975), and at this level of analysis there is general agreement: the relative response rate approximates the relative reinforcement rate (for a recent review see de Villiers, 1977). However, there is no similar consensus about the relationship between individual response and reinforcement probabilities in *conc VI VI* schedules. Shimp (1969), Mackintosh (1974), and others (e.g., Silberberg and Williams, 1974) have suggested that subjects maximize the momentary reinforcement probabilities and choose the alternative that has the highest expected value at the moment.

At a molecular level, maximizing of momentary reinforcement probabilities results in sequential dependencies between responses; at a molar level, this strategy is said (Shimp, 1969) to produce matching between overall relative response frequencies and overall relative reinforcement frequencies. In other words, the momentary maximizing theory states that matching, a relationship between averaged measures, is a secondary byproduct of a molecular-level optimizing process. However, in two discrete-trial choice procedure studies, (Herrnstein, 1971; Nevin, 1969; and see de Villiers, 1977), response sequences did not appear to follow the pattern predicted by the momentary maximizing hypothesis. In fact, some of the data suggested that the probability of switching from one reinforcement alternative to the other, a changeover, did not vary as a function of previous responding. The apparent absence of sequential dependencies suggested that a simple Markov chain model might fully describe molecular-level performance in *conc VI VI* schedules.

In a *conc VI VI* schedule, reinforcement probabilities change from moment to moment. While the subject responds at one schedule, the probability that a reinforcer is available at the other schedule increases. The momentary maximizing theory, therefore, predicts that the probability of a switch from one schedule to the other should increase as a

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function of the number of responses since the last switch. Alternatively, a simple Markov chain model predicts that the probability of switching from a schedule will not vary as a function of previous responding. The relationship between switching and previous responding is described in the study reported here.

Figure 1 shows the two Markov models that were tested. The upper-case letters identify states: *R* corresponds to responding at one reinforcement schedule; *G* corresponds to responding at the other reinforcement schedule. The lower-case letters stand for the transition or changeover probabilities. The top diagram represents a first-order Markov model (Bishop, Fienberg, and Holland, 1975) of *conc VI VI* performance. It indicates that the probability of a changeover at each chance to switch, that is, after a response on a schedule-associated manipulandum, depends only on the schedule to which the subject is currently responding. The first-order model prediction, then, is that

the response-by-response changeover probabilities are *stationary*; in other words, that the changeover probabilities are constant and independent of the number of responses since the last changeover. The bottom diagram represents a second-order Markov model of *conc VI VI* performance. The terms R_1 and G_1 stand for the first postchangeover responses (states). The terms R_{2+} and G_{2+} stand for all subsequent postchangeover responses (states), at the respective schedules, which start at the second postchangeover response and continue until the next changeover). Therefore, for the second-order model, changeover probabilities are stationary following the first postchangeover response. Thus, according to the second-order model, the probability of a changeover depends on two factors: first, which of the two reinforcement schedules the subject is responding to, and, second, whether the last response was a changeover response. These are the two simplest Markov models of concurrent performance possible.

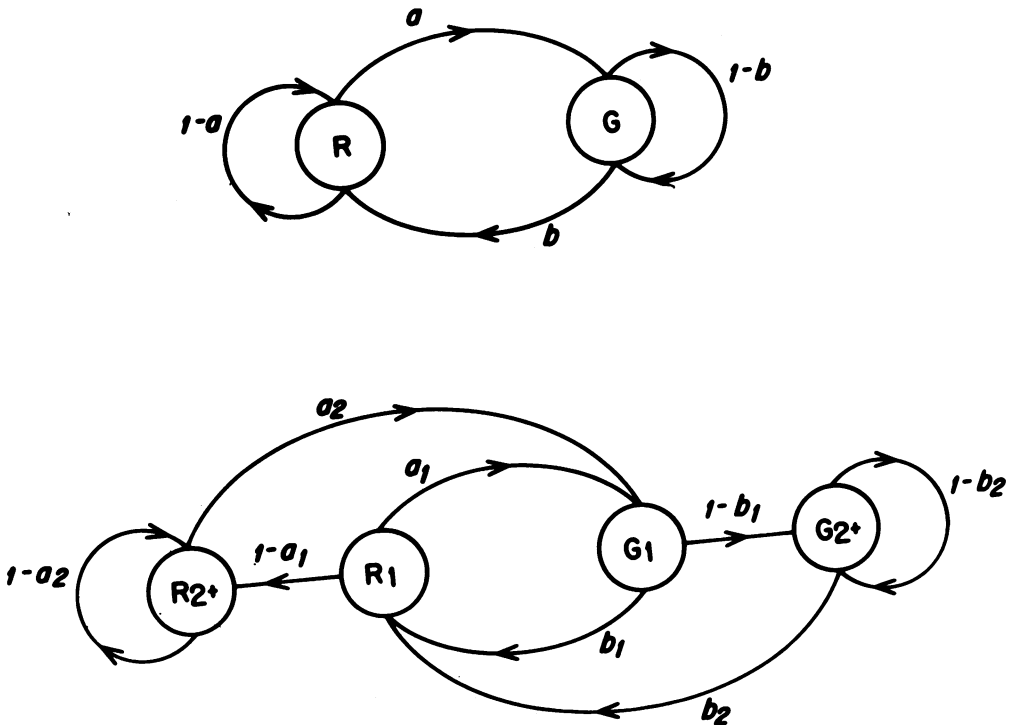


Fig. 1. The top diagram shows a first-order Markov process description of *conc VI VI* performance. The upper-case letters indicate responding at the two reinforcement schedules: *R* for the schedule associated with the red stimulus, *G* for the schedule associated with the green stimulus. The lower-case letters stand for the response probabilities. The bottom diagram is a second-order Markov process description of *conc VI VI* performance. The terms R_1 and G_1 stand for the first postchangeover responses; R_{2+} and G_{2+} stand for all subsequent postchangeover responses. The lower-case letters stand for the response probabilities associated with each state.

METHOD

Subjects

Four White Carneaux pigeons without previous experimental histories were maintained at 80% of their free-feeding weights.

Apparatus

A standard chamber, 31.0 cm high, 33.0 cm deep, and 29.5 cm wide, housed the experiment. The response keys (Gerbrands) were 1.9 cm in diameter, 22.0 cm from the floor, and 14.5 cm apart. A force of more than 0.15 N operated the keys, and each effective response produced a brief feedback click and a brief flicker of the illuminated response keys. The opening of the grain hopper was 8.9 cm from the floor, midway between the two keys. Mixed grain was delivered for 2.5 sec with a standard feeder (Gerbrands), which was illuminated by two 7-W lamps during reinforcement. The experimental chamber was enclosed in a sound-attenuating box and lit by two 28-V dc lamps. White noise masked extraneous sounds, and a computer (Digital Equipment Corporation PDP-9T) controlled the presentation of stimuli and recording of experimental events.

Procedure

Reinforcers were scheduled by a changeover-key *conc* VI VI procedure (Findley, 1958). The right key, designated the main key, was associated with both VI schedules; the left key, designated the changeover key, controlled which of the two schedules was available at the main key. The main key was illuminated red for one VI schedule and green for the other, and pecks on this key intermittently produced grain. The changeover key was illuminated white. A single peck on the changeover key alternated the color of the main key and the available VI schedule. Although only one schedule at a time was available, both ran concurrently.

Reinforcers were scheduled so that their relative rate was constant (Stubbs and Pliskoff, 1969). First, a single VI 30-sec schedule determined when a reinforcer was available. The intervals, based on the list provided by Fleshler and Hoffman (1962), gave an approximately exponential distribution of scheduled interreinforcement times. Second, when an interval timed out, a binary digit drawn at random determined whether the reinforcer was as-

signed to the red or green stimulus. For example, if the probabilities of assigning a reinforcer were 0.75 and 0.25 for the two stimuli, the scheduled interreinforcement intervals were 40 sec and 120 sec respectively.

There was, however, one important change from the standard concurrent procedure. To simplify interpretation, a changeover-delay procedure was not used. That is, the first main-key response following a changeover-key response could be reinforced independently of time. Successive pecks at the changeover key, however, had no effect, so that a changeover-key response was necessarily preceded by a main-key response.

Before exposure to the concurrent schedules, the birds were trained to peck the response keys according to an autoshaping procedure (Brown and Jenkins, 1968). Each bird was then exposed to three different concurrent schedules: *conc* VI 40-sec VI 120-sec, *conc* VI 60-sec VI 60-sec, and *conc* VI 300-sec VI 33.3-sec (see Table 1). Each schedule pair was maintained until both the relative response rate and the overall average probability of a changeover (the ratio of total changeovers to total responses) did not show a trend for five sessions (an extreme value). Changeover probability has not been considered a criterion for stability by other researchers, but if this measure were not stable, then the response-by-response (molecular) changeover probabilities could not be stable.

Sessions were terminated after 60 reinforcers or 40 min. The experiment was conducted six days a week.

RESULTS

Molar Measures

Table 1 summarizes the overall performance measures, based on data averaged from the last five sessions of each condition. Columns list the following information: number of sessions in each condition, response rates for the red and green stimuli, time spent responding at the red and green stimuli, exclusive of reinforcement time, changeover rates, and reinforcement rates for the two stimuli.

Figure 2 shows relative response frequency (left panels) and relative time (right panels) as a function of relative reinforcement rate for the schedules associated with the red stimu-

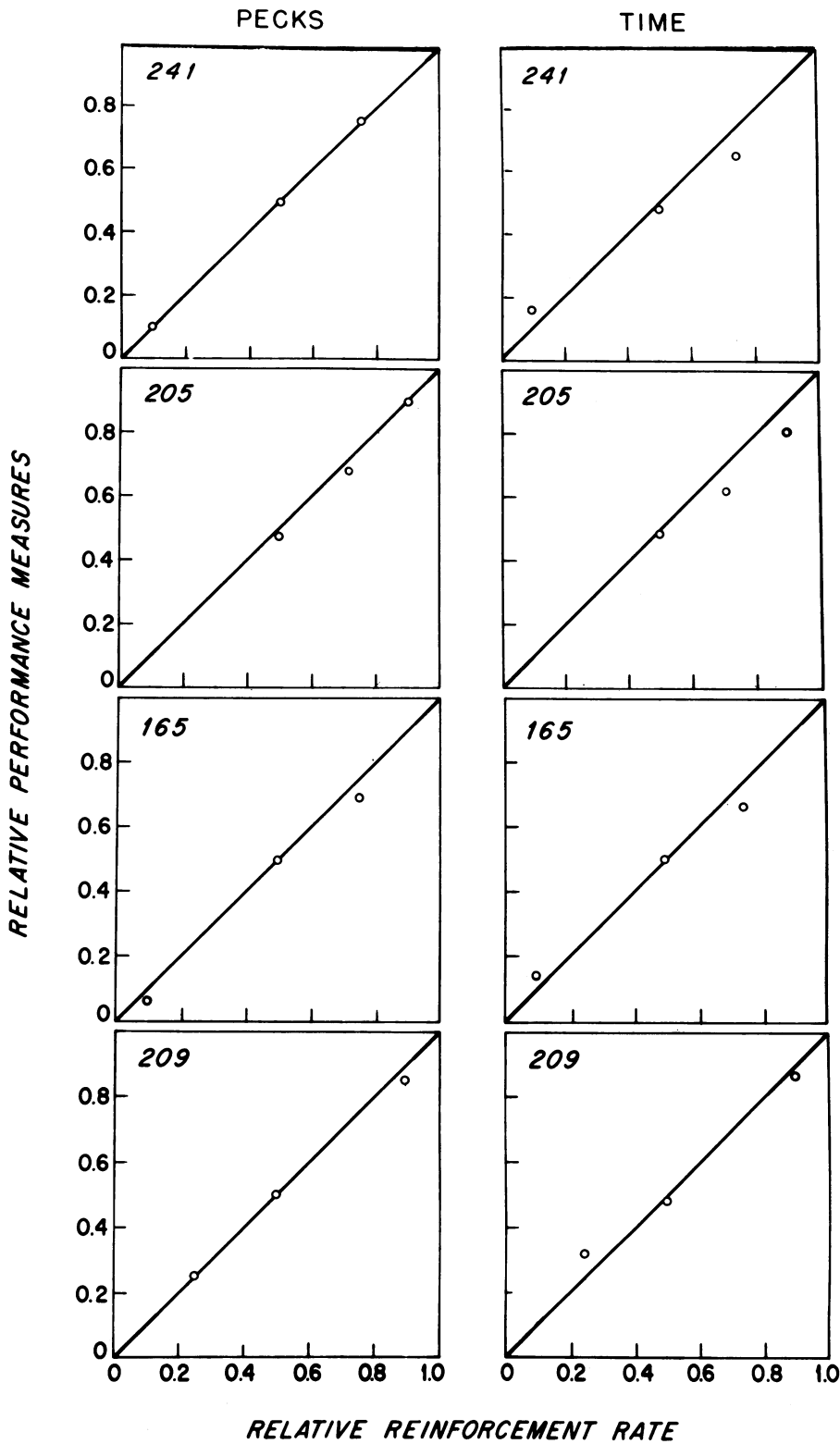


Fig. 2. Relative response rate (left) and relative time (right) plotted as a function of relative reinforcement rate. The data were pooled from the last five sessions of each condition.

Table 1

Summary of the results, based on data averaged from the last five sessions of each condition. Standard deviations are enclosed in parentheses.

Subject Number	Variable Intervals		Session Number	Pecks per Minute		Time (min)		Changeovers per Minute	Obtained Rein- forcements per Hour	
	(sec)			Red	Green	Red	Green		Red	Green
	Red	Green								
165	40.0	120.0	75-117	21.04 (3.03)	9.40 (2.65)	24.08 (4.78)	12.85 (3.77)	7.69 (2.17)	64.98	25.99
	300.0	33.3	1-32	4.86 (1.00)	74.46 (1.76)	4.92 (1.35)	29.22 (2.97)	4.60 (0.53)	10.55	93.15
	60.0	60.0	33-74	13.38 (1.40)	13.47 (1.81)	17.23 (1.12)	17.07 (0.95)	17.11 (2.04)	54.23	50.73
209	120.0	40.0	1-40	18.06 (1.05)	54.06 (2.22)	10.12 (0.95)	21.38 (0.32)	18.06 (1.87)	26.67	87.62
	33.3	300.0	41-85	72.42 (5.27)	12.64 (2.08)	27.12 (1.07)	4.60 (0.55)	7.35 (0.66)	102.15	11.35
	60.0	60.0	86-165	42.13 (3.74)	42.07 (4.01)	15.22 (0.88)	15.73 (0.68)	23.17 (1.71)	56.22	58.16
241	40.0	120.0	73-132	58.31 (2.48)	18.24 (2.19)	20.70 (0.53)	10.77 (0.57)	22.72 (0.64)	87.70	26.69
	300.0	33.3	44-72	6.70 (1.02)	59.26 (3.90)	5.27 (0.72)	26.23 (0.97)	11.11 (0.79)	13.33	100.94
	60.0	60.0	1-43	24.83 (1.79)	24.52 (1.79)	15.85 (0.45)	15.97 (0.20)	28.41 (2.01)	58.46	54.69
205	40.0	120.0	1-38	33.63 (3.22)	15.74 (0.92)	19.72 (0.53)	11.98 (0.38)	22.30 (0.95)	87.07	24.50
	33.3	300.0	39-97	60.00 (3.46)	6.66 (0.72)	26.18 (1.43)	5.93 (0.58)	8.50 (0.78)	100.88	11.21
	60.0	60.0	98-130	28.26 (2.35)	31.14 (1.49)	15.30 (0.45)	16.23 (0.20)	35.87 (2.73)	57.08	57.08

lus. Despite the absence of a changeover delay, relative pecks closely matched relative reinforcement rate. The largest difference between the two relative measures was 9%, and the slope of the best-fitting line (least squares) for

the group relative peck frequencies was 0.95. Relative time did not fit the diagonal indicating perfect matching as closely. The largest deviation was 15%, and the slope of the best-fitting line for the group time data was 0.77.

VI 40 sec and VI 120 sec SCHEDULES

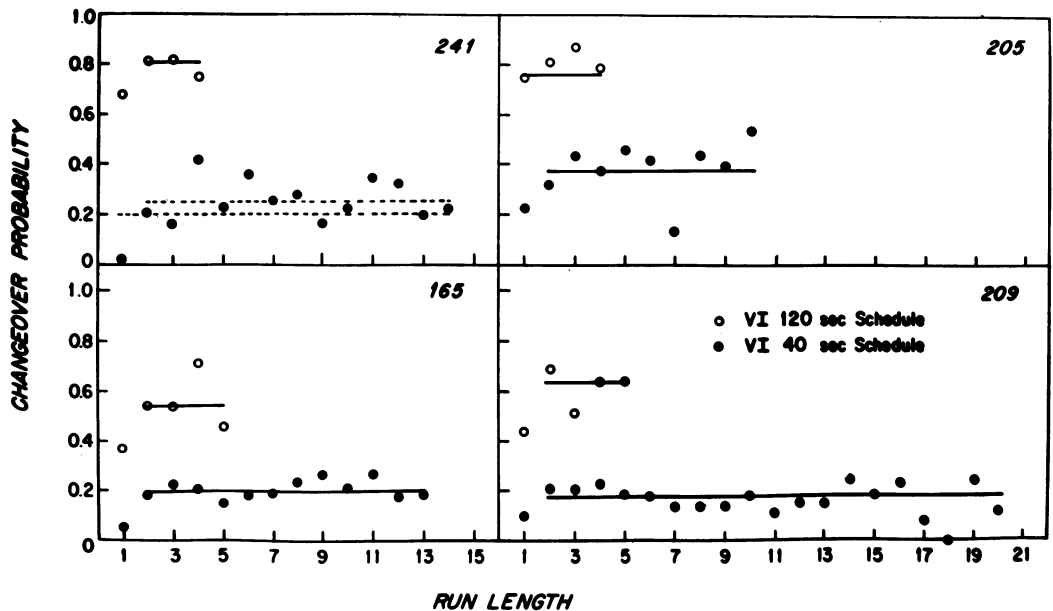


Fig. 3. The probability of a changeover-key response as a function of the number of responses since the last changeover (run length). The data are from the last session of the VI 40-sec VI 120-sec condition. The horizontal lines indicate the changeover probabilities predicted by the Markov models. When the first-order model provided the best fit, the horizontal line starts at the first postchangeover response; when the second-order model provided the best fit, the horizontal line starts at the second postchangeover response. The broken horizontal lines indicate the first- and second-order model predictions when it was not possible to fit either of the two models to the data.

Molecular Measures

Figures 3, 4, and 5 show the conditional probability of a changeover-key response as a function of the number of responses since the last changeover (run length). The open circles indicate changeover probabilities from the schedule with the lower reinforcement rate;

filled circles indicate these probabilities for the schedule with the higher reinforcement rate. For the *conc* VI 60-sec VI 60-sec schedule, the filled circles show the probabilities for the schedule associated with the red stimulus. The data are from a single session, the last one in each condition.

The conditional changeover response proba-

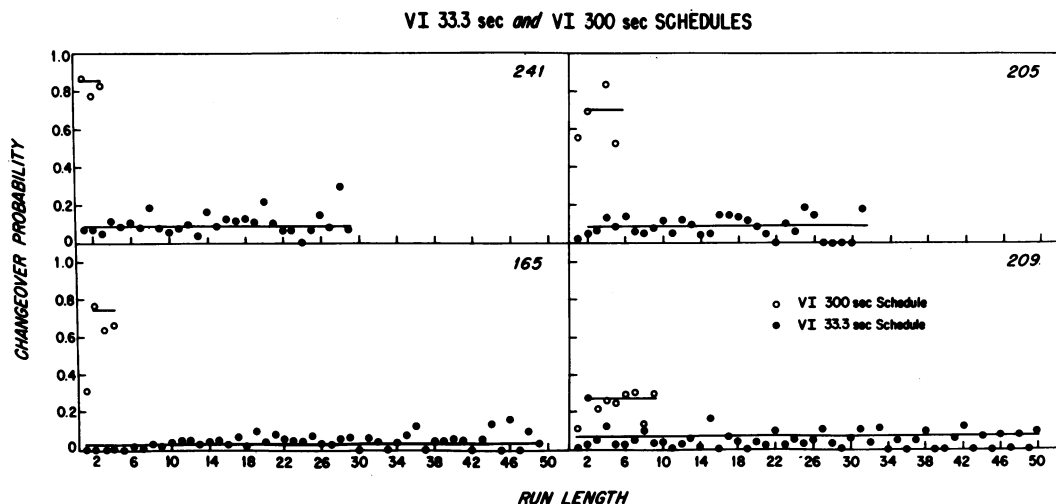


Fig. 4. The probability of a changeover-key response as a function of the number of responses since the last changeover (run length). The data are from the last session of the VI 33.3-sec VI 300-sec condition. The horizontal lines indicate the Markov model predictions. See Figure 3 and text for further discussion of the Markov model predictions.

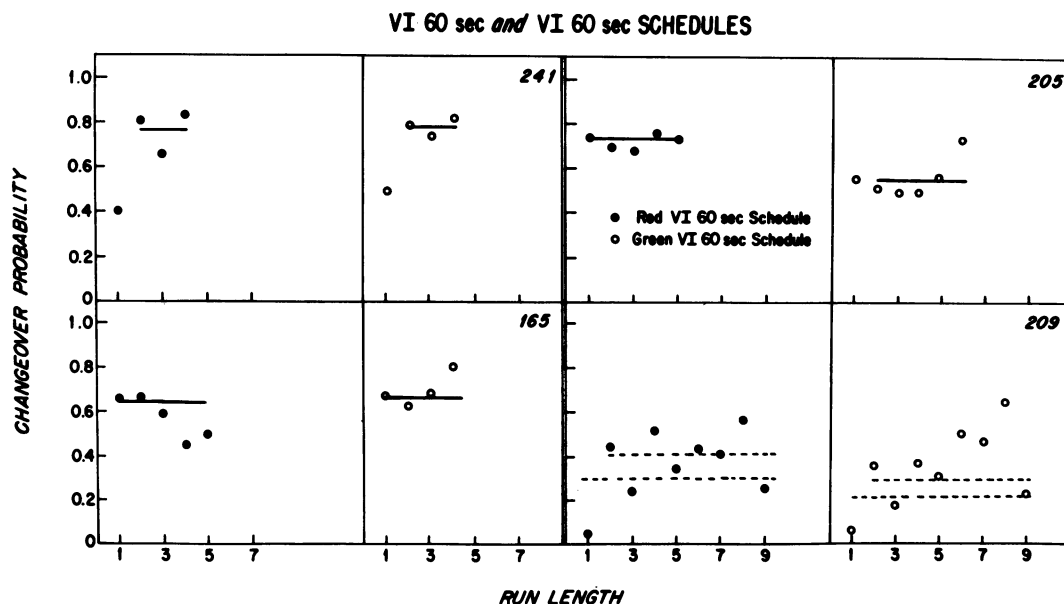


Fig. 5. The probability of a changeover-key response as a function of the number of responses since the last changeover (run length). The data are from the last session of the VI 60-sec VI 60-sec condition. The horizontal lines indicate the Markov model predictions. See Figure 3 and text for further discussion of the Markov model predictions.

bilities were calculated from the number of opportunities to changeover at each run length. For example, the number of opportunities to switch after a run of two main-key responses is the number of runs that are two pecks long and longer. Of necessity, the number of opportunities to switch must decrease as run length increases. When the number of opportunities to switch at a run length was less than 10, the remaining run lengths were grouped, and the probability of a changeover-key response was calculated from these data. This probability is shown with the longest run length.

As noted above, the momentary maximizing hypothesis leads to the prediction that changeover probabilities in Figures 3, 4, and 5 should increase as a function of run length. However, a Kendall nonparametric trend test (Ferguson, 1965) showed that changeover probabilities did not monotonically increase (or decrease) as run length increased. The molecular level events predicted by the momentary maximizing hypothesis, then, did not appear to occur.

The Markov account shown in Figure 1 requires stationary changeover probabilities from the first or second postchangeover response. A goodness-of-fit test (chi-square) was used to test for stationarity.

To test the first-order Markov model, it is necessary to determine if the individual, response-by-response changeover probabilities generally approximated their average (which is simply the ratio of the number of changeovers from a schedule to the number of responses on the schedule). To fit the second-order Markov model, it is necessary to show that, starting from the second postchangeover response, the individual, response-by-response changeover probabilities generally approximated their average.

Cochran (1954) proposed a simple way to strengthen the chi-square test for goodness of fit. Set the criterion for collapsing adjacent classes at an expected frequency of less than one, rather than at the customary less than five. This rule increases the likelihood of rejecting the Markov models, and for this reason it was adopted. However, it turned out that, except for longer runs on the VI 33.3-sec schedule, the expected changeover frequencies were almost always greater than five.

Table 2 lists the results of the chi-square analyses. The Markov models were accepted

at $p > 0.05$ (Bishop *et al.*, 1975). By this criterion, the first-order model described the response-by-response behavior in 16 of 24 tests, and the second-order model described the performance in 20 of 23 tests. (For Pigeon 241 on the VI 300-sec schedule there were not enough degrees of freedom to test the second-order Markov model). In general, then, the response-by-response probabilities of switching from a schedule were constant and independent of the number of previous responses since the last switch.

The straight lines in Figures 3, 4, and 5 show the changeover probabilities predicted by the Markov models. The lines that start at the first postchangeover response indicate the average probability of switching from a schedule, which is the first-order model prediction; the lines that start at the second postchangeover response indicate the average probability of switching for runs of two and longer, which is the second-order model prediction. The postchangeover response at which the lines start tells whether the first- or second-order model provided the best fit (higher value of p). The broken lines show the predicted changeover probabilities for the three sets of data that the Markov models did not fit ($p \leq 0.05$). For these data, both the first- and second-order model predictions are displayed.

Changeover probabilities showed a cyclic pattern in the sessions in which they were not stationary. For example, the probabilities of switching at even-numbered run lengths were always greater than the probabilities at the two adjacent, odd-numbered run lengths for Pigeon 209 on the *conc* VI 60-sec VI 60-sec schedule. (A similar pattern is shown by Pigeon 241 on the VI 40-sec schedule.) Odd-even cycles suggest two peck "bursts", a characteristic of pigeons that has been reported elsewhere (*e.g.*, Blough, 1966).

DISCUSSION

The primary finding was that simple first- and second-order Markov models described the response-by-response performance of four pigeons on several *conc* VI VI schedules. This result means that the observed molecular response structure was not controlled by the molecular reinforcement contingencies, that matching was not a secondary byproduct of a molecular level maximizing strategy, and that

Table 2

Summary of the chi-square test results. The data are from the last session of each condition. First-order is for the test that included the first postchangeover response. Second-order is for the test that excluded the first postchangeover response. The degrees of freedom correspond to two less than the longest run on the schedule, except in those instances in which adjacent classes (run lengths) were collapsed. One degree of freedom was lost because the expected changeover probability, the average, was estimated from the data and one degree was lost for the last class. The p values give an estimate of the probability of the chi-square sum, and the higher the p the smaller the difference between the individual changeover probabilities and the average (predicted) changeover probability. The third column shows the session relative response frequency.

<i>Subject Number</i>	<i>Variable Intervals (sec)</i>	<i>Per Cent Pecks to Red</i>	<i>Markov Order</i>	<i>D.F.</i>	<i>Chi-Square</i>	<i>p</i>
241	40.0	77.5%	1	12	134.08	<0.01
			2	11	45.91	<0.01
	120.0		1	2	2.16	>0.30
			2	1	0.002	>0.95
	33.3	9%	1	26	30.64	>0.20
			2	25	29.04	>0.20
	300.0		1	1	0.18	>0.50
			2	-	-	-
	60.0 (Red)	52.2%	1	2	60.75	<0.01
			2	1	1.37	>0.20
	60.0 (Green)		1	2	24.38	<0.01
			2	1	0.12	>0.70
205	40.0	70%	1	8	27.65	<0.01
			2	7	9.70	>0.20
	120.0		1	2	0.78	>0.50
			2	1	0.15	>0.50
	33.3	88.6%	1	27	38.58	>0.05
			2	26	27.49	>0.30
	300.0		1	2	3.33	>0.10
			2	1	1.26	>0.20
	60.0 (Red)	46.3%	1	3	0.46	>0.90
			2	2	0.11	>0.90
	60.0 (Green)		1	4	3.38	>0.30
			2	3	0.40	>0.90
165	40.0	72.7%	1	11	16.22	>0.10
			2	10	3.08	>0.95
	120.0		1	3	6.25	>0.10
			2	2	0.93	>0.50
	33.3	6.5%	1	38	50.41	>0.05
			2	37	50.47	>0.05
	300.0		1	2	16.85	<0.01
			2	1	0.29	>0.50
	60.0 (Red)	50.9%	1	3	0.92	>0.80
			2	2	0.87	>0.50
	60.0 (Green)		1	2	0.60	>0.70
			2	1	0.57	>0.30
209	40.0	24%	1	18	25.40	>0.10
			2	17	11.75	>0.80
	120.0		1	3	13.30	<0.01
			2	2	2.01	>0.30
	33.3	86%	1	41	52.50	>0.10
			2	40	51.59	>0.10
	300.0		1	7	10.95	>0.05
			2	6	2.68	>0.80
	60.0 (Red)	46.8%	1	7	136.0	<0.01
			2	6	21.30	<0.01
	60.0 (Green)		1	7	115.21	<0.01
			2	6	31.78	<0.01

independent of run length, changeover probabilities were proportional to the overall relative frequencies of reinforcement.

For a Markov process, the transition probabilities between states are proportional to the relative frequency of time (or the number of trials) the states are occupied. Therefore, since the Markov models fit the data, the probability of a switch to a schedule, independently of run length, must have been proportional to the relative frequency of responding at the schedule. And, because the relative frequency of responding approximated the relative frequency of reinforcement (Figure 2), the probability of switching to a schedule at any run length must also have been approximately proportional to the overall relative frequency of reinforcement obtained there. These interrelationships can be shown explicitly by the following mathematical relations inherent to *conc VI VI* schedules and Markov processes.

In a *conc VI VI* schedule, the average probability of switching to a reinforcement alternative is directly proportional to the relative frequency of responding at that alternative:

$$\frac{C_1}{B_1} \left(\frac{B_1 B_2}{B_1 C_2 + B_2 C_1} \right) = \frac{B_2}{B_1 + B_2}$$

and

$$\frac{C_2}{B_2} \left(\frac{B_1 B_2}{B_1 C_2 + B_2 C_1} \right) = \frac{B_1}{B_1 + B_2}, \quad (1)$$

where B_i is the total number of responses at alternative i , C_i is the total number of changeovers from alternative i , and since the number of changeovers from the two schedules cannot differ by more than one, it is assumed that $C_1 = C_2$. The first quotient in each expression is the average probability of switching from alternative i . The second quotient is the constant of proportionality that links the average changeover probabilities to the relative response frequencies. Its magnitude is equal to one-half the harmonic mean of the interchangeover times (Heyman, 1977), so that it scales the overall tendency to switch. Since the chi-square analyses showed in general that the response-by-response changeover probabilities did not differ from their overall average, the probability of switching at any run length can be substituted for the overall average probability of switching in Equation 1. Figure 2 shows that relative response frequency approximated relative reinforcement frequency. Therefore, rela-

tive reinforcement frequency can be substituted for relative response frequency in Equation 1. These substitutions result in the corresponding approximations:

$$p(x|b_1, n) \left(\frac{B_1 B_2}{B_1 C_2 + B_2 C_1} \right) \cong \frac{R_2}{R_1 + R_2}$$

and

$$p(x|b_2, n) \left(\frac{B_1 B_2}{B_1 C_2 + B_2 C_1} \right) \cong \frac{R_1}{R_1 + R_2}, \quad (2)$$

where b_i , n is a main-key postchangeover response of run length n , x is a changeover response, and R_i is the total number of reinforcements at alternative i . The two equations show that independent of run length, the probability of a changeover to a schedule was approximately proportional to the relative reinforcement frequency provided by the schedule. Whenever the first-order model fits, the approximations hold for all run lengths ($n = 1, 2, \dots, n$). Whenever the second-order model fits but the first does not, the approximations hold for runs of two and longer ($n = 2, 3, \dots, n$).

When the first-order Markov model did not describe the data, factors other than relative reinforcement frequency must have influenced switching. Figures 3, 4, and 5 suggest what these factors might be. First, for sessions in which a second-order model fit, but a first did not, the probability of a changeover at a run length of one was typically lower than the probability of a changeover for longer runs. This difference may have been due to the topographical asymmetry between first and later postchangeover responses. The first postchangeover response followed a response at the changeover key, which was 14.5 cm from the main key; subsequent postchangeover responses simply followed one another at the main key. The probability of switching at a run length of one, then, may have been reduced because it followed a more effortful response. Second, the cyclic, odd-even changeover probability fluctuations produced deviations from the first-order model. Two-peck response units would explain this pattern, and it has been suggested (Blough, 1966) that pigeons may have an innate tendency to "double peck". In any event, neither of these departures from the first-order Markov model suggests the influence of changes in molecular-level reinforcement probabilities.

There is some evidence that stationary changeover probabilities are a general characteristic of *conc VI VI* responding when a changeover delay is not employed, and in procedures that use a changeover delay, there is indirect evidence that switching probabilities are approximately stationary once the delay has elapsed.

Herrnstein (1971) recorded changeover probabilities as a function of run length in a discrete-trial version of a *conc VI VI* schedule (the data are displayed in de Villiers, 1977). A changeover delay was not used, and by inspection the changeover probabilities appear stationary. Of additional significance is that Herrnstein arranged reinforcers with two independent timers, rather than with the single timer (Stubbs and Pliskoff, 1969) procedure used here. The similarity of outcomes suggests that the molecular structure of performance in the two types of *conc VI VI* procedures is the same despite differences in the molecular reinforcement contingencies and differences in the relationship between overall responding and overall reinforcement rate (Heyman, 1977).

In procedures that use a changeover delay, switching probabilities cannot be strictly stationary. That is, a first-order Markov model would not apply, because subjects appear to learn not to switch during the delay interval (Baum and Rachlin, 1969). However, the relationship between absolute changeover rate and relative response frequencies in *conc VI VI* performance suggests that postdelay changeover probabilities are approximately stationary, and postdelay responding takes up the bulk of the session. Absolute changeover rate decreases as relative response frequency diverges from 0.5 (e.g., Baum, 1973; Herrnstein, 1961). For a Markov process, the function relating switching to preference is obtained by considering a series of first-order models in which the transition probabilities, a and b in Figure 1, differ from model to model but add to a constant. For example, if a plus b sum to one, the first-order model is equivalent to a Bernoulli trials process, and the expected probability of a switch at each trial is $2p(1-p)$. When $2p(1-p)$ is graphed as a function of p (see Mosteller, Rourke, and Thomas, 1970, p. 213), which is analogous to the common practice of showing changeover rate as a function of, say, relative peck frequencies, the slope is negatively accelerated with a maximum at

$p = 1/2$. Similarly in *conc VI VI* performance, changeover rate shows a negatively accelerated increase as relative response frequency approaches 0.5. Moreover, the negatively accelerated changeover function appears to be a general characteristic of *conc VI VI* responding: it is obtained with different species, for example humans (Baum, 1975), rats (Baum, 1973), and pigeons (Stubbs and Pliskoff, 1969), it is obtained in procedures that arrange reinforcers with a single timer (Stubbs and Pliskoff, 1969) or two independent timers (Herrnstein, 1961); and it is obtained independently of a changeover delay (changeover functions calculated from Table 1 show a negatively accelerated slope and a maximum at about $p = 0.5$). In other words, a basic feature of *conc VI VI* performance can be derived from the assumption that changeover probabilities are generally stationary.

The molar results of this study are also of some theoretical interest because it is widely believed that matching depends on the use of a changeover delay (e.g., Mackintosh, 1974; Shimp, 1975). Figure 2, however, shows response matching well within the range obtained in studies using a changeover delay (de Villiers, 1977). Additionally, several other studies have omitted a changeover delay yet obtained acceptable matching (Baum, 1974; Bradshaw, Szabadi, and Bevan, 1976; Findley, 1958; Stubbs and Pliskoff, 1969); reviews have ignored these results.

Since the probability of reinforcement at the unattended schedule increases as a function of run length, it is, perhaps, surprising that the changeover probabilities were, in general, stationary. However, consider the view that the overall distribution of reinforcements directly determines the overall distribution of behavior (de Villiers and Herrnstein, 1976). This account of *conc VI VI* performance is strongly supported by the finding that independent of run length, the response-by-response changeover probabilities were proportional to the overall relative frequencies of reinforcement (Equation 2). Moreover, the results that could not be described by a first-order Markov model do not appear to contradict the molar account, since fluctuations in changeover probabilities were apparently due to factors dissociated from the schedules; for example, a tendency for pigeons to peck in bursts. In sum, the simple orderliness of molecular-level responding

makes it possible to describe fully the sequential history of about 35 min of responding, typically about 2400 responses, with models that entailed at most four parameters, and, as is true of molar-level concurrent responding, molecular-level behavior showed a simple symmetry with the overall relative reinforcement frequencies.

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